$$\cos\varphi\sin(C+\omega)=\sin\omega[-(x/a)\cos A+(y/b)\cos(\omega-B)]$$
$$\sin\varphi\sin(C+\omega)=\sin\omega[(x/a)\sin A-(y/b)\sin(\omega-B)].$$

Squaring and adding, and transposing the members of the equation, we obtain the required equation of the locus

$$\frac{x^2}{a^2}+\frac{2xy}{ab}\cos(C+\omega)+\frac{y^2}{b^2}=\frac{\sin^2(C+\omega)}{\sin^2\omega}.$$

The locus is therefore an ellipse with center at the intersection of the two lines.

Solved similarly by *G. W. GREENWOOD*, and *G. B. M. ZERR*.

CALCULUS.

163. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Can there be a plane curve the length of which varies *directly as the abscissa* and *inversely as the ordinate* of any point on the curve ?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$S=mx/y,\ dS=\frac{mydx-mxdy}{y^2}=\sqrt{(dx^2+dy^2)}.$$

$\therefore\ \sqrt{[1+(dx/dy)^2]}=[my(dx/dy)-mx]/y^2.$
Let $x=vy$, $dx/dy=v+y(dv/dy)=v+yp$, suppose.
$\therefore\ \sqrt{[1+(v+yp)^2]}=mp$, or $v+yp=\sqrt{(m^2p^2-1)}$.....(1).

From (1), $p=\dfrac{vy\pm\sqrt{(v^2m^2+m^2-y^2)}}{m^2-y^2}=\dfrac{xy\pm\sqrt{[m^2(x^2+y^2)-y^4]}}{y(m^2-y^2)}.$

Differentiating (1) with respect to y,

$$dv/dy=p=p--y(dp/dy)+\frac{m^2p(dp/dy)}{\sqrt{(m^2p^2-1)}}.$$

$$\therefore\ 2p=\left(\frac{m^2p}{\sqrt{(m^2p^2-1)}}-y\right)\frac{dp}{dy}.$$

$$\therefore\ \frac{dy}{dp}=\frac{m^2}{2\sqrt{(m^2p^2-1)}}-\frac{y}{2p}\ \text{or}\ \frac{dy}{dp}+\frac{y}{2p}=\frac{m^2}{2\sqrt{(m^2p^2-1)}}.$$

$$\therefore\ y\sqrt{p}=C+\frac{m^2}{2}\int\frac{\sqrt{(p)}dp}{\sqrt{(m^2p^2-1)}}=C+\frac{m^2}{2}f(p)\text{.....(2)}.$$

The value of p in (2) from (1) gives the primitive, which is the curve desired. By series we get

$$\frac{m^2}{2}\int\frac{\sqrt{(p)}dp}{\sqrt{(m^2p^2-1)}}=\frac{1}{2\sqrt{p}}\sqrt{(m^2p^2-1)}+\tfrac{1}{2}m\sqrt{(p)}(1+\frac{1}{2.3m^2p^2}+\frac{1}{2.4.7\,m^4\,p^4}$$

$$+\frac{3}{2.4.6.11\,m^6p^6}+\frac{3.5}{2.4.6.8.15m^8p^8}+\dots).$$

164. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $m^2+n^2=1$, $m^2\cos^2\theta+n^2\cos^2\varphi=A$, $a^2b^2\sin^2\theta(m^2+n^2\cos^2\varphi)+a^2c^2\cos^2\theta\cos^2\varphi$ $+b^2c^2\sin^2\varphi(n^2+m^2\cos^2\theta)=B$, $\sqrt{(1-m^2\sin^2\theta)}=\triangle(\theta)$, $\sqrt{(1-n^2\sin^2\varphi)}=\triangle(\varphi)$ prove that $\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{\triangle(\theta)\triangle(\varphi)}=(\frac{1}{6}\pi)(a^2b^2+a^2c^2+b^2c^2)$.

Solution by the PROPOSER.

$x^2/a^2+y^2/b^2+z^2/c^2=1$.

$$\int ds=\int\int\sqrt{[1+(dz/dx)^2+(dz/dy)^2]}dxdy=c^2\int\int\frac{\sqrt{[z^2/c^4+x^2/a^4+y^2/b^4]}dxdy}{z}.$$

Let p=perpendicular from center on tangent plane.

$\therefore\ 1/p=\sqrt{[z^2/c^4+x^2/a^4+y^2/b^4]}$.

Let $(b/a)\sqrt{[a^2-x^2]}=y'$, $(a/b)\sqrt{[b^2-y^2]}=x'$.

$$D=\int\frac{ds}{p}=c^2\int\int\frac{[z^2/c^4+x^2/a^4+y^2/b^4]dxdy}{z}=\frac{1}{c^2}\int_0^a\int_0^{y'}zdxdy+\frac{c^2}{a^4}\int_0^a\int_0^{y'}\frac{x^2}{z}dxdy$$

$$+\frac{c^2}{b^4}\int_0^b\int_0^{x'}(y^2/z)dydx=\frac{1}{abc}\int_0^a\int_0^{y'}\sqrt{[a^2b^2-b^2x^2-a^2y^2]}dxdy$$

$$+\frac{abc}{a^4}\int_0^a\int_0^{y'}\frac{x^2dxdy}{\sqrt{[a^2b^2-b^2x^2-a^2y^2]}}+\frac{abc}{b^4}\int_0^b\int_0^{x'}\frac{y^2dydx}{\sqrt{[a^2b^2-b^2x^2-a^2y^2]}}$$

$$=\tfrac{1}{6}\pi(ab/c+bc/a+ac/b)=\frac{\pi}{6abc}(a^2b^2+b^2c^2+a^2c^2).$$

Let $x=a\sin\varphi\sqrt{(1-m^2\sin^2\theta)}$, $y=b\cos\theta\cos\varphi$, $z=c\sin\theta\sqrt{(1-n^2\sin^2\varphi)}$, $m^2+n^2=1$; $dx/d\varphi=a\cos\varphi\sqrt{(1-m^2\sin^2\theta)}$, $dx/d\theta=-\dfrac{am^2\sin\varphi\sin\theta\cos\theta}{\sqrt{(1-m^2\sin^2\theta)}}$, $dy/d\varphi=$ $-b\cos\theta\sin\varphi$, $dy/d\theta=-b\sin\theta\cos\varphi$; $dxdy=(dx/d\theta)(dy/d\varphi)-(dx/d\varphi)(dy/d\theta)$